Applications of Bayesian Networks to Cost Engineering for Satellites

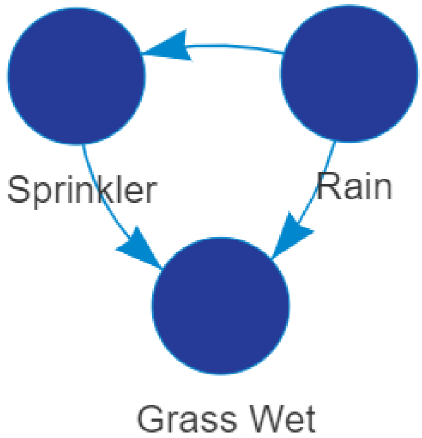
September 16, 2021

Overview

- Bayesian Networks Overview
- Learn the Network Structure
- Fit the Bayesian Network
- Querying and Making Predictions
- Some Examples
- Questions

Bayesian Networks Overview

- Bayesian networks are a probabilistic graphical model that encodes the dependency structure between variables
- Learning or specifying the network structure between variables results in a Directed Acyclic Graph (DAG)
  - Visualizing the DAG illustrates the relations between the circular nodes (i.e., variables) and edges (i.e., the arrows that illustrate the direction of conditional dependence)

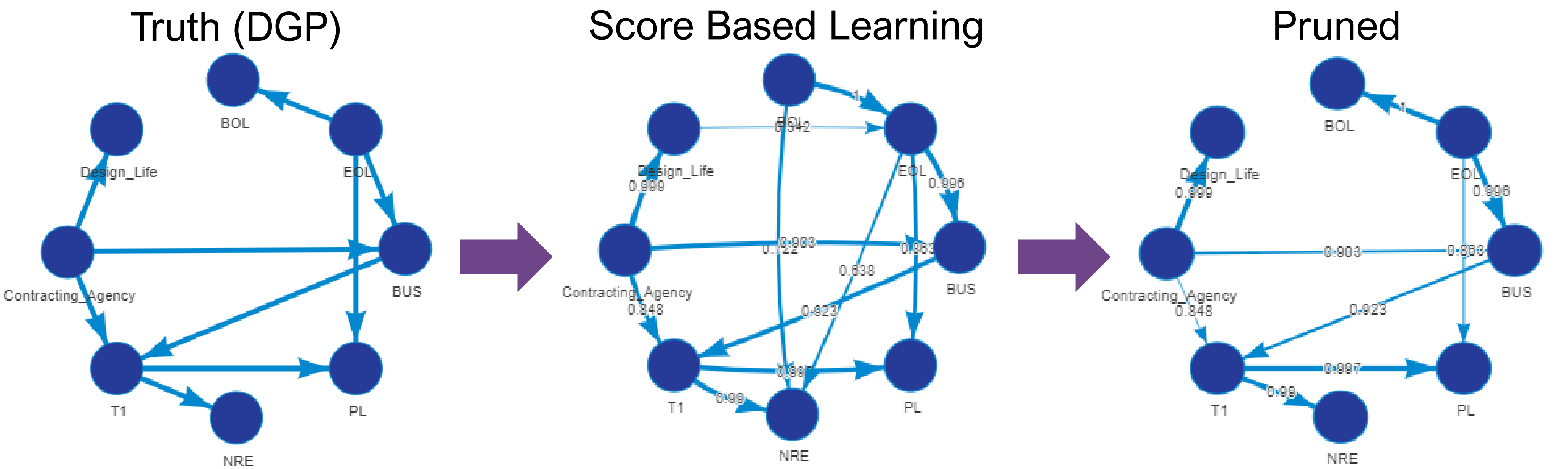


- For categorical variables, each node encodes a conditional probability table, and for numeric variables, each node encodes the conditional (e.g., gaussian) distribution
- With a Bayesian Network in hand, we can formally ask probabilistic questions of our data based on what we know

Learn the Network Structure

- Network Structure Learning Algorithms (Learning Edges)
  - Constraint Based Methods
    - Determine conditional independence between variables with statistical tests (e.g., log-likelihood ratio test, Pearson's χ^2 test, etc.)
    - Constraint-based methods tend to work better at smaller sample sizes but tend to take more time to learn
  - Score Based Methods
    - Assign a score (e.g., log-likelihood, BIC, etc.) to candidate DAG structures, and select the optimal network
  - To evaluate the sensitivity of the edges learned to our data, we can apply a network learning algorithm to bootstrapped samples of our data and then calculate the proportion of learned networks that contain a particular edge between nodes
    - We can then prune edges that fall below a given threshold
  - Apply SME Judgement add, remove or reorient edges

Learn the Network Structure



- Score Based Network Learning (optimizing BIC)
  - The network learning algorithm identified three non-existent edges
- Pruned Edge Strengths
  - For an arbitrary number of bootstrap samples, we learn the network structure on each bootstrap sample and record the edges between nodes; the values on each edge in the visualizations above represent the proportion of bootstrap samples whose learned network contained that particular edge
  - In this case, pruning away the lower strength edges allows us to perfectly replicate (i.e., learn) the true data generating process

Fit the Bayesian Network

- Having learned the network (i.e., conditional independence) structure, we can determine the conditional probability tables and conditional distributions that define the network
- Parameters can either be estimated with frequentist (i.e., Maximum Likelihood) or Bayesian (i.e., expected value of the posterior) approaches
- Gaussian Bayesian networks assume normality of conditionally independent variables, and residual normality for conditionally dependent variables (which are estimated with linear regression)
  - In cases when we would expect a particular variable or the residuals of a particular conditional distribution to depart from normality, the standard remedies would apply (e.g., variable transformations or discretization into a multinomial distribution)

Fit the Bayesian Network

$$BOL = 0.97 + 0.9 * EOL$$

$$\left.\begin{matrix} N(5.0, 0.15), if\ Commercial \\ N(4.7, 0.53), if\ NonCommercial \end{matrix}\right\} = DL$$

$$EOL = N(\mu = 7.8, \sigma = 1.2)$$

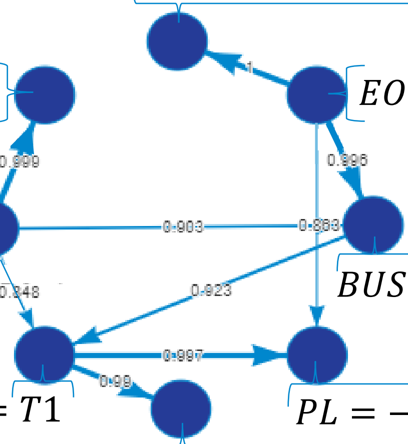
$$\left.\begin{matrix} 48\%\ Commercial \\ 52\%\ NonCommercial \end{matrix}\right\} = CA$$

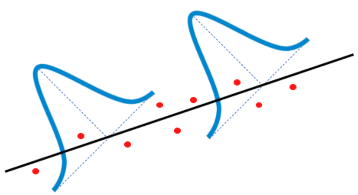
$$BUS = \begin{cases} 2.1 + 0.6 * EOL, if\ Commercial \\ 4.7 + 0.4 * EOL, if\ NonCommercial \end{cases}$$

$$\left.\begin{matrix} 4.3 + 0.95 * BUS, if\ Commercial \\ 4.6 + 0.98 * BUS, if\ NonCommercial \end{matrix}\right\} = T1$$

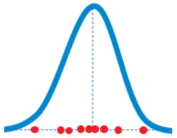
$$PL = -2.2 + 0.5 * T1 + 0.4 * EOL$$

$$NRE = -7.5 + 1.6 * T1$$

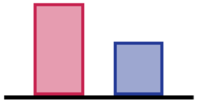




Gaussian Distribution centered about regression line, with standard deviation equal to the residual standard error: BOL, BUS, PL, NRE, T1



Gaussian Distribution centered around the sample mean, with standard deviation equal to the sample standard deviation: EOL and DL



Conditional Probability Table for Multinomial Distribution: Contracting Agency (CA)

Querying and Making Predictions

- Having fitted a Bayesian network, we can now formally ask (i.e., query) probabilistic questions of the data, and make predictions for new data points, given the information we have
  - Querying the Network
    - To query the network, we specify evidence (i.e., things we know, or suspect are true), and ask for the likelihood of a particular event
      - For example, we can ask the following question: "What is the likelihood of [Event] $NRE \leq 12$ and $10 < T1 \leq 11$, given [Evidence] $BUS + PL \leq 14$?" – Answer: 39.1%
  - Making Predictions
    - To make predictions, we specify evidence and ask the network to impute missing information
      - For example, given [Evidence] $EOL = 6.6$, what are reasonable values for the remaining 8 variables?
      - Answer: Bus = 6.9, PL = 5.9, NRE = 10.4, T1 = 11.2, CA = Non-Commercial, DL = 4.8, BOL = 6.9

Some Examples

Shiny web-app for Bayesian Networks

Tool Demo

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